Quantification of the conditional value of SHM data for the fatigue safety evaluation of a road viaduct

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ABSTRACT:

Fatigue safety verification of existing bridges that uses ‘‘re-calculation’’ based on codes, usually results in insufficient fatigue safety, triggering invasive interventions. Instead of “re-calculation”, Structural Health Monitoring (SHM) should be used for the assessment of the existing bridges. Monitoring systems provide data that can reduce uncertainties associated with the fatigue loading process and the structural resistance. The objective of this paper is to quantify the value of the SHM system implemented in a 60-years-old road viaduct to investigate its fatigue safety, through modeling of the fundamental decisions of performing monitoring in conjunction with its expected utility. The quantification of the conditional value of information is based on the decision tree analysis that considers the structural reliability, various decision scenarios as well as the cost-benefit assessments. This leads to a quantitative decision basis for the owner about how much time and money can be saved while the viaduct fulfills its function reliably and respects the safety requirements. The originality of this paper stands in the application of the value of information theory to an existing viaduct considering the fatigue failure of the system based on the monitoring data and the cost-benefit of monitoring method.

INTRODUCTION

The fatigue assessment of existing bridges is important for sustainable use from both technical and economical point of view. To achieve this, bridge managers should understand existing bridges and use tools to take accountable decisions about their current and future fatigue safety. Bridge assessment based on re-calculations using design code provisions usually results in insufficient fatigue safety that requires strengthening or replacing the structure. This finding is often a problem on paper only and does not reflect the real performance of existing bridges. Subsequently, and in order to make the best decisions during the assessment, structural health monitoring (SHM) system is used, and the value of SHM data is quantified based on the decision tree that considers the structural reliability, various decision scenarios as well as the cost-benefit assessments. This methodology is illustrated with a case study, Crêt de l'Anneau Viaduct.

The structure is a 60-year-old composite concrete-steel road-viaduct (Figure 1) located in Switzerland, as part of a cantonal road leading from Switzerland to the French border. It has seven typical spans of 25.6m length and an approach span of 15.8m. The reinforced-concrete (RC) slab of variable thickness ranging from 17 to 24cm is fixed on two steel girders of 1.3m height. The girder is composed of a series of single span beams linked by hinges. The total length of the viaduct is 195m.

Because of a "re-calculation" based on design code provisions, the viaduct was suspected to present fatigue problems after 60 years of service. To take the best decision about doing nothing or replacing the structure, SHM system was implemented in the viaduct in June 2016 to investigate its effective fatigue behavior. For such a situation, a value of information analysis can be utilized to quantify the value of performing SHM and to derive the optimal decision about doing nothing or replacing the structure.

The Value of Information (VoI) theory has been developed by Raiffa and Schlaifer (1961) and is rooted in Bayesian updating and utility-based decision theory with a specific format to quantify the utility increase due to additional information. The utility increase of additional and already obtained information is termed as the Conditional Value of Sample Information (CSVI).

Monitoring system

A Structural Health Monitoring (SHM) system is implemented for one year to investigate the fatigue behavior of the viaduct. More details about the monitoring system can be found in (Bayane and Brühwiler 2018).Two techniques are used including strain gauges to measure the strain in steel reinforcement bars and thermocouples to measure the temperature of the concrete, the steel, and the air. Two slabs are instrumented, and for each slab, strain gauges are implemented in two transverse rebars and two longitudinal rebars at the mid-span, which is the most loaded part of the RC slab.

Figure 1: View of the Crêt de l'Anneau Viaduct

Monitoring data

The most critical part of the viaduct for fatigue is the RC slab since the recorded strains in the steel girder are smaller than the endurance limit. As such, the fatigue verification of the viaduct for the case study is focused on the RC slab in which the fatigue failure is determined by the failure of the steel rebars.

Stress cycles are calculated from the annual measured strain in the most loaded rebars. Temporal variation of stresses is first deduced from the recorded strain by a multiplication with the steel elastic modulus of 210GPa. The Rainflow counting method is then used to provide a set of stress cycles from stress variations. Table 1 presents the stress cycles $n_{i,1}$ over one year for each stress range i of the instrumented transverse rebar 1 at the mid-span, for the recorded stresses $\Delta\sigma_{i,1}$.

Table 1. Stress spectra

$\Delta\sigma_{i,1}$ [MPa]	$n_{i,1}$
5	67051
10	18180
15	6391
20	2744
25	1091
30	392
35	181
40	75
45	24
50	12
55	5
60	2
65	3
70	1
85	1

PROBABILISTIC MODEL FOR THE STRUCTURAL SYSTEM

The probability of fatigue failure of the viaduct is evaluated before the monitoring, using code provision criteria, which is given in the Swiss Standard, and after the monitoring, using the recorded data. Therefore, two probabilistic models are developed, the prior model corresponding to code provisions and the posterior model corresponding to monitoring data. The formulation of the fatigue limit state for the prior and posterior models will be based on the S-N curve approach.

Fatigue safety is verified according to two levels; the first level requires that the fatigue action effect is below the endurance limit. The second level is performed when the first level is not fulfilled. It requires the calculation of damage accumulation according to Miner's rule where the total fatigue damage must be less than 1.

To perform fatigue damage accumulation, the S-N curve parameters are taken from the Swiss standard SIA269. The straight reinforcement bars of the viaduct have a fatigue resistance $\Delta\sigma_{sd,fat}$ equal to 150 MPa, and an endurance limit $\Delta\sigma_{s,D}$ of 120 MPa. The slope m of the S-N curve is equal to 4 (SIA269).

Prior damage model

Based on the S-N curve, with Miner's accumulation rule, the fatigue limit of rebar j in the concrete can be expressed by $g_j(t)$ (Thöns, 2018):

$$g_j(t) = \Delta - D_{prior_j}(t) \tag{1}$$

$$D_{prior_j}(t) = n_D t \frac{E[\Delta\sigma_D{}^m]}{K} \tag{2}$$

$$E[\Delta\sigma_D{}^m] = (M_L M_\sigma M_D M_K k)^m \Gamma(1 + \frac{m}{\lambda}) \tag{3}$$

Γ is the gamma function, $\Delta\sigma_D$ is the design value of stresses that has a Weibull distribution (Thöns et al. (2015)) with the parameters λ and k, which are the scale and the location parameters. K is the material parameter from the S-N curve, m is the slope value, n_D is the annual cycle. M_L is the model uncertainty of traffic load. M_σ is the model uncertainty of stress ranges. M_D is the model uncertainty of accumulated damage. M_K is the model uncertainty of S-N curve. The parameters λ and k of the stress distribution are adjusted to reach both the mean value of $\Delta\sigma_D$ which is equal to $E(\Delta\sigma_D)$ and an accumulated fatigue damage of 1.0 after the service life t_{SL}, i.e.120 years.

$$\lambda * \left(\Gamma\left(1 + \frac{1}{k}\right)\right) = E(\Delta\sigma_D) \tag{4}$$

$$\frac{n_D t_{SL} \, (M_L M_\sigma M_D M_K k)^m \Gamma\left(1+\frac{m}{\lambda}\right)}{K} = 1 \tag{5}$$

Table 2 includes the random variables, their distributions and their parameters used to perform the prior study. Monte Carlo simulation is used to find the cumulative probability of component failure throughout the service duration.

Table 2. Probabilistic model for the random variables, prior study

Var.	Des.	Dist.	Mean	Std.	Ref.
$\Delta\sigma_D$	*Design value of stresses [MPa]*	*WB*	*200*	-	*FEM SIA 261*
Δ	*Miner's sum at failure*	*LN*	*1.0*	*0.3*	*JCSS*
n_D	*Annual cycles [/year]*	*Det.*	*7.10⁵*	-	*SIA 261*
m	*Slope value*	*Det.*	*4*	-	*SIA 269*
K	*Material parameter from SN curve [MPa]*	*LN*	10^{15}	*0.58*	*SIA 269 & JCSS*
k	*Location parameter*	*Det.*	*Cali.*	-	*Eq. 4,5*
λ	*Scale parameter*	*Det.*	*Cali.*	-	*Eq. 4,5*
M_L	*Uncertainties related to traffic load calculation*	*LN*	*0.68*	*0.102*	*Folsø et.al. (2002)*
M_σ	*Uncertainties related to stress calculation*	*LN*	*1.00*	*0.05*	*Folsø et.al. (2002)*
M_D	*Uncertainties related to accumulated damage*	*LN*	*1.00*	*0.05*	*JCSS, for rebar*
M_K	*Uncertainties related to S-N curve*	*LN*	*1.00*	*0.05*	*Assumed*

The annual cycles of heavy trucks for principal roads is equal to 350'000 cycles per direction. This value was taken from the European traffic and was reduced by 30% to consider the volume of traffic in Switzerland (SIA261).

The recalculation value of stresses $\Delta\sigma_D$ was obtained using the load model 1 presented in the Swiss Standards (SIA261). The load model was applied to a 3D Finite Element Model (FEM) of the viaduct, considering the initial properties of materials and boundary conditions. The maximum stress at the mid transverse span of the slab was calculated and multiplied by a load factor of 1.50 to determine the re-calculation value of stress of 200 MPa (SIA261).

The prior fatigue damage of the instrumented rebar was calculated according to Eq. 1-5. A normal distribution $f_{D_{prior}}$ was fitted to the prior damage, and the corresponding mean and standard deviation were identified. The prior damage distribution is plotted in Figure 3.

Posterior damage model

Monitoring data provides the stress range and the corresponding cycles. The fatigue safety is then evaluated according to the level one of verification. Since the highest recorded stress range of 85 MPa is significantly smaller than the endurance limit (120MPa), the level one of fatigue verification is fulfilled as illustrated in Figure 2. Therefore, to perform a Miner's damage calculation, an arbitrarily chosen amplification factor of 4 is applied such that the stress ranges exceed the endurance limit and the fatigue damage can be calculated.

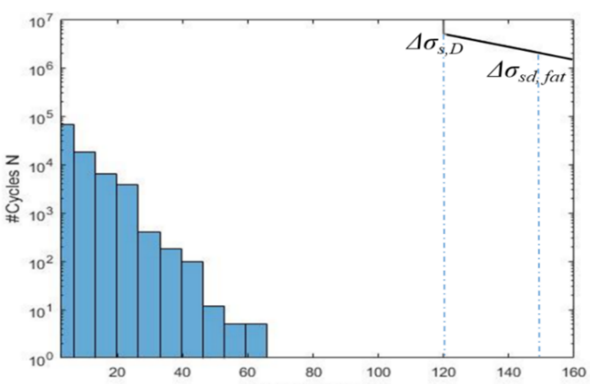

Figure 2. Annual stress ranges and cycles of the most loaded rebar

A likelihood damage model is developed based on the recommendations of JCSS (2006). Like for the prior study the model uses the S-N approach that can be expressed in the form of:

$$N\Delta\sigma^m = k \quad (6)$$

where N is the number of stress cycles to failure at a constant amplitude stress range $\Delta\sigma$, and k and m are material parameters.

In order to deal with variable amplitude loading in the S-N approach, fatigue damage is quantified in terms of Miner's damage summation. According to this rule, all stress cycles cause proportional fatigue damage, which is linearly additive. The scatter in the stress history may be neglected, and the damage D_{like_j} of the rebar j is equal to:

$$D_{like_j} = \sum_i \frac{n_{i,j}}{N_{i,j}} \quad (7)$$

where $N_{i,j}$ is the number of stress cycles to failure at a constant amplitude stress range $\Delta\sigma_{i,j}$ and $n_{i,j}$ is the number of actual stress cycles for the stress range $\Delta\sigma_{i,j}$

with $\quad logN_{i,j} = logk - mlog\Delta\sigma_{i,j} + \varepsilon \quad (8)$

where ε is the statistical error in the SN curve

and $\quad \Delta\sigma_{i,j} = E(\Delta\varepsilon_{i,j} + M_\varepsilon) \quad (9)$

where E is the young modulus of steel rebars
$\Delta\varepsilon_{i,j}$ is the strain range i for the rebar j
M_ε is the measurement error

The likelihood of damage can then be written as follow:

$$D_{like_j} = \sum_{\Delta\sigma_{i,j} > \Delta\sigma s,D} \frac{n_{i,j} t\left(E(\Delta\varepsilon_{i,j} + M_\varepsilon)\right)^m}{10^{\varepsilon + logk}} \quad (10)$$

Table 3 includes the definition of the random variables, their distributions and their parameters used to calculate the likelihood damage. Monte Carlo simulation is used to find the cumulative probability of component failure throughout the service duration.

Table 3. Probabilistic model for the random variables, likelihood study

Var.	Des.	Dist.	Mean	Std.	Ref.
Δ	*Miner's sum at failure*	*LN*	*1.0*	*0.3*	*JCSS*
ε	*Statistical error in SN curve*	*N*	*0*	*0.5*	*JCSS*
E	*Young modulus of steel (MPa)*	*LN*	*2.1 10^5*	*0.05*	*JCSS*
M_ε	*Monitoring error*	*N*	*0*	*10^{-6}*	*Monitoring*
Log k1	*Normal (MPa)*	*N*	*16. 2862*	*0.4*	*(Rastayest, et al., 2018)*
m	*Slope value*	*Det.*	*4*	*-*	*SIA 269 (Swiss standard)*

A normal distribution $f_{D_{like}}$ was fitted to the likelihood damage and the resultant mean, and the standard deviation is calculated.

The likelihood damage distribution for the instrumented rebar is plotted in Figure 3.

Based on Bayesian updating theory, the posterior damage distribution $f_{D_{post}}$ can be updated as:

$$f_{D_{post}}\left(d_{post}\right) = \frac{f_{D_{prior}}\left(d_{prior}\right) \cdot f_{D_{like}}\left(d_{like}\right)}{c} \qquad (11)$$

where c is a constant ensuring the integral of the posterior density function equals 1.0, and d is the realization of (prior, likelihood or posterior) damage.

The posterior damage also has a normal distribution. The mean and standard deviation of the posterior model are identified accordingly. The normalized probability density function of the prior and the posterior damages and the likelihood are presented in Figure 3. The posterior damage follows the same shape of the likelihood, and it is far away from the prior damage. Therefore, the information provided by the likelihood is considered in the rest of the study as being the posterior information.

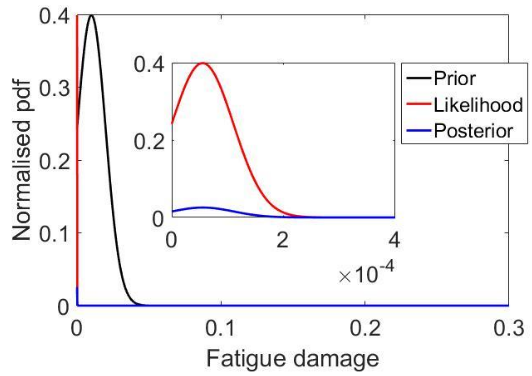

Figure 3. Fatigue damage distribution (prior, likelihood, and posterior)

The limit state function for the posterior model of a component j is written as:

$$g_j(t) = \Delta - \sum_{\Delta\sigma_{i,j} > \Delta\sigma_{s,D}} \frac{n_{i,j} t\left(E(\Delta\varepsilon_{i,j} + M_\varepsilon)\right)^m}{10^{\varepsilon + logk}} \qquad (12)$$

Probability of failure of the system

The system fatigue failure of the viaduct is modeled. The viaduct system is of series type with different subsystems. The system failure is dominated by the weakest subsystem, which is the slender slab of 17cm thickness. From monitoring data, the cyclic stresses recorded in the transverse cross section were two times higher than in the longitudinal section. Therefore, the fatigue failure of the viaduct is assumed equal to the fatigue failure of the cross-section of the reinforced-concrete slab.

Herwig (2008), Johanssor (2004), and Schläfli and Brühwiler (1997) have shown that the fatigue failure of the reinforced concrete slabs is due to the failure of the rebars. The fracture of an isolated rebar (inside the concrete) may be considered as brittle; however, with the distributed reinforcement (254 rebars for the case of study), the failure of the cross-section has the potential for fatigue ductile behavior (Herwig, 2008). Consequently, the slab is modelled as a ductile Daniels system consisting of 254 components. The limit state function for the system is then presented in Eq. 13:

$$g_{sys}(\mathrm{t}) = \sum_{j=1}^{254} \left(\Delta - D_j(t)\right) \qquad (13)$$

Monitoring data is available for the most loaded rebar located at mid-span. The distribution of the strain in all the rebars is taken from the finite element model of the structure. Fatigue stresses decrease linearly with a factor of 0.0008/rebar when moving from mid-span toward the box girders. The stress of each rebar j is then calculated according to Eq. 14:

$$\Delta\sigma_{i,j} = (1 - (j - 1) * 0.0008) * \Delta\sigma_{i,1} \qquad (14)$$

where $\Delta\sigma_{i,j}$ is the stress range i of the rebar j, and $\Delta\sigma_{i,1}$ corresponds to the stress range i of the instrumented rebar 1. The cumulative probability of failure of the system is equal to:

$$P(F_S(t)) = P(g_{sys}(t) \leq 0) \qquad (15)$$

It is calculated using both the prior and posterior models. The prior and posterior cumulative probabilities of the system failure are shown as:

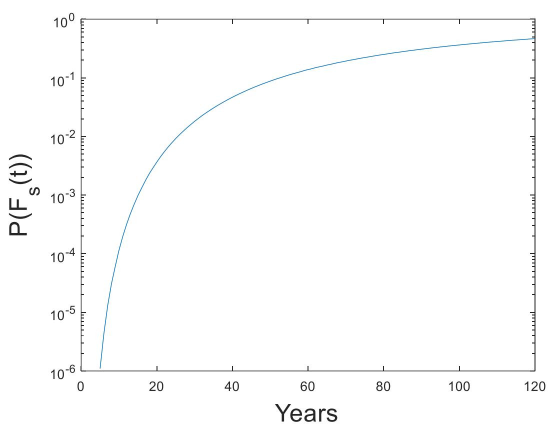

Figure 4. Prior cumulative probability of system failure

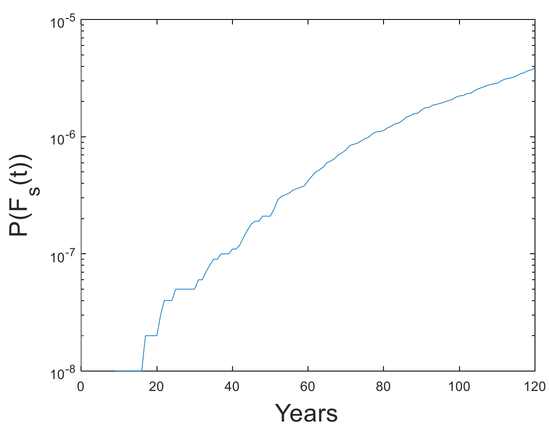

Figure 5. Posterior cumulative probability of system failure

The probability of failure calculated based on monitoring data is small, even after amplifying the loads by a factor of four, and assuming that the past traffic was similar to the present traffic. The heavy trucks are not frequent on the viaduct, and the slab is well reinforced, which explains the low-recorded strain values and the small probability of failure.

After 60 years of service, the prior probability failure is 0.172. According to the JCSS (2006), the target probability of failure is chosen as 5×10^{-4} for the existing bridge as the relative costs for safety measures are large and the consequences of failure are moderate. For the case study, the target probability of failure is exceeded according to the prior model but not reached for the posterior model.

CONDITIONAL VALUE OF SAMPLE INFORMATION ANALYSIS

The viaduct manager has to make decision about which action to take depending on the states of the viaduct namely to do nothing or to replace. The viaduct manager can reach the decision based on the minimum expected costs without additional information, which is modelled with a prior decision analysis or by considering the already obtained additional information. The latter decision can be modeled with a posterior decision analysis. With the difference of minimum expected costs for both cases (with and without additional information) and with the consideration uncertainties related to the additional information, a conditional value of sample information can be calculated (CSVI according Raiffa and Schlaifer (1961)).

The decision process can be described as shown in Figure 6 with a_i denoting the choice of the actions. θ_i is the viaduct states which can be safe or failure. e_i represent the different information of strategies. z_i is the outcome of the strategies. In this case, the information of z_1, no fatigue problem, is obtained after monitoring. We use u_i to present the expected utilities regards different actions under different strategy information, which is calculated by multiplying the probabilities and the consequences. Here we only consider the cost, so that the choice of action is performed based on the minimized expected costs.

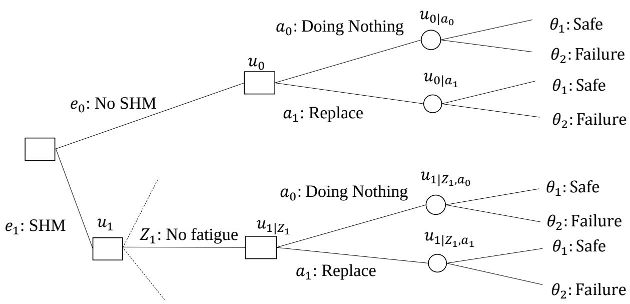

Figure 6. Illustration of decision tree

The conditional value of sample information is calculated as:

$$CVSI = u_0 - u_{1|z_1} \quad (16)$$

$$u_{1|z_1} = min[u_{1|z_1,a_0}; u_{1|z_1,a_1}] \quad (17)$$

$$u_0 = min[u_{0|a_0}; u_{0|a_1}] \quad (18)$$

$$r(F_S(t)) = \frac{dP(F_S(t))/dt}{1-P(F_S(t))} \quad (19)$$

$$u_{1|z_1,a_0} = \sum_{t=1}^{T_{SL}} r\left(F_S(t)|Z_{1_{t_m}}\right) C_F \frac{1}{(1+\gamma)^t} + C_M \quad (20)$$

$$u_{1|z_1,a_1} = \sum_{t=1}^{T_{SL}} r\left(F_S(t)|R_{t_m}, Z_{1_{t_m}}\right) C_F \frac{1}{(1+\gamma)^t} + C_M + C_R \quad (21)$$

$$u_{0|a_0} = \sum_{t=1}^{T_{SL}} r(F_S(t)\,) C_F \frac{1}{(1+\gamma)^t} \quad (22)$$

$$u_{0|a_1} = \sum_{t=1}^{T_{SL}} r\left(F_S(t)|R_{t_m}\right) C_F \frac{1}{(1+\gamma)^t} + C_R \quad (23)$$

$r(F_S(t))$ is the prior annual probability of failure. $r\left(F_S(t)|Z_{1_{t_m}}\right)$ is the posterior annual probability of failure given indication of no fatigue after monitoring. $r\left(F_S(t)|R_{t_m}\right)$ is the annual probability of failure after replacing the viaduct at year t_m based on prior knowledge. $r\left(F_S(t)|R_{t_m}, Z_{1_{t_m}}\right)$ is the annual probability of failure after obtaining the indication of no fatigue information and replacing the viaduct at year t_m. In this case $t_m = 60$ year and service life $T_{SL} = 120$ years. The replacement would result in a new viaduct.

The cost model is shown in Table 4. Since the height of the viaduct is from 2 to 7 meters, it can lead rarely to death in the case of failure. Considering the extreme case, the cost of failure is assumed to be equal to the cost of one person's life due to the collapse of the viaduct given in the Swiss Standards.

Table 4. Cost model

Cost	*Categories*	*Value*	*Reference*
C_R	*New structure (Replace)*	*5.5 MCHF*	*Assumed*
C_M	*Monitoring (for one year)*	*40 kCHF*	*Real case study*
C_F	*Cost of failure*	*10 MCHF*	*SIA 269*
γ	*Discounting factor*	*0.02*	*Higuchi(2008)*

Based on Eq. 16-23 and Table 4, the calculation of utilities results is shown as:

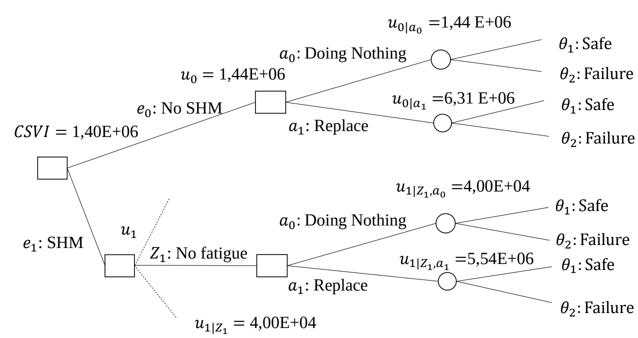

Figure 7. Decision tree with expected utilities

In Figure 7 it is shown that, without the structural health monitoring data, i.e. only with information provided by re-calculations based on codes, the viaduct has a very high probability of fatigue failure. Due to the associated high and unacceptable risks, the viaduct would be required to be replaced (action a_1). With monitoring data, action a_0 (do nothing) would be preferable due to the lower expected utilities. Thus, the Conditional Value of Sample Information is 1.4 MCHF, which means that by spending 40 kCHF money for monitoring, 1.4 million CHF of the cost is saved while keeping the viaduct in service.

CONCLUSIONS

The presented case study shows that the monitoring approach is found to give valuable information about the evaluation of the fatigue safety of the viaduct. The results show that there is no fatigue problem in the viaduct even by amplifying the monitored fatigue stresses arbitrarily by a factor of 4.

Through quantifying the conditional value of SHM information for this viaduct, by modeling the fatigue failure of the cross reinforced-concrete slab as a system failure, it is found that the money has been saved, the risk can be reduced and that the viaduct can operate much longer. It is demonstrated how SHM information can be utilized to support the optimal decision for a continuous monitoring, by integrating sound scientific structural models, SHM engineering models and cost and consequence models.

The SHM results indicate a significant bias of the model uncertainty in the design models. This indication may be used to derive models for value of information analyses with not yet obtained SHM information in order to predict for which bridges a SHM analysis may be valuable. This would support a quantitative decision basis for the owner based on an optimization of the time and money for keeping bridges reliably fulfilling their functions and being safe for users.

ACKNOWLEDGEMENTS

This research work was performed within the European project INFRASTAR (infrastar.eu), which has received funding from the European Union’s Horizon 2020 research and innovation program under the Marie Skłodowska-Curie grant agreement No 676139. The grant is gratefully acknowledged.

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